More about Wake Fields

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Kick Factors are not enough

$\Delta y' = Ky$

Good for

– Looking at how position jitter becomes angular jitter

– Comparison between measurement and theory

But

– K depends on component *and* pulse shape

– Only includes lowest order (dipole) term. Not enough to describe what happens near the wall

– Does not describe change in pulse shape (emittance, banana bunches)

Mesh methods too heavy for tracking

MAFIA, ECHO, HFSS etc solve Maxwell's equations on a grid for a given current distribution

See last talk/next talk for details/progress

Too slow for a tracking code!

Take two particles...

Work with Wake Potentials $\vec{W} = \int ds(\vec{E} + \vec{v} \times \vec{B})$ with integration along particle's undisturbed trajectory (rigid beam approx.)

Consider leading particle at r', ϕ, s, effecting trailing particle at $r, \phi, 0$.

For axial symmetry wake potential can be expanded as

$$\vec{W}_\perp = -e \sum_{m=1} W_m(s) m r^{m-1} r'^m (\hat{r} cos(m\theta) - \hat{\theta} sin(m\theta))$$

where $\theta = \phi' - \phi$

Z & K Eq 3.20

Also $\vec{W}_{||} = -e \sum_{m=0} (1 + \delta_0^m) W'_m(s) r^m r'^m cosm\theta$

Z & K Eq 3.2

Take many+1 Particles

EM fields add. Effect of many particles in a slice on some trailing particle is just given by adding/integrating.

Need to use $cosm(\phi' - \phi) = cos(m\phi)cos(m\phi') + sin(m\phi)sin(m\phi')$ etc

$$\vec{W}_\perp = -e \sum_{m=1} W_m(s) m r^{m-1} [(\hat{r}(Q_m cos(m\phi) + \tilde{Q}_m sin(m\phi)) - \hat{\theta}(Q_m sin(m\phi) - \tilde{Q}_m cos(m\phi))]$$

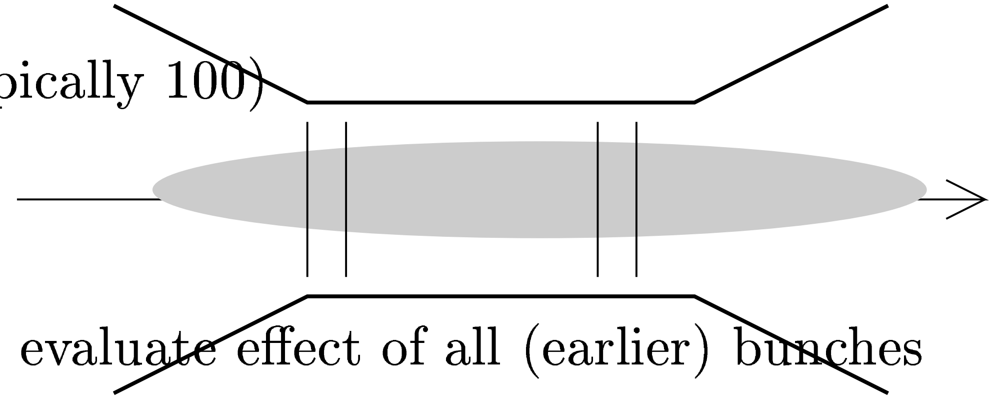
Q_m and $\tilde{Q}_m$ are $\sum_i r_i'^m cos(m\phi_i')$ and $\sum_i r_i'^m sin(m\phi_i')$ Depend (only) on properties of leading slice

$W_m(s)$ - wake functions - depend only on component

r, ϕ depend only on trailing particle being affected.

Merlin approach

Divide bunch into slices (typically 100)



For each particle in bunch j evaluate effect of all (earlier) bunches

$\Delta p_\perp = \sum_i Q_i W(s_i - s_j)$

Q_i = charge centroid of leading bunch i

$W(s)$ is a member function of the component. Supplied by user. Evaluated only $100 \times 100/2$ times. (With small correction based on gradient.) Can be resistive or geometric (or both.)

This is radial part of the lowest-order ($m = 1$) term from previous slide.

Extending Merlin

Include $\hat{\theta}$ effect

Add more terms (we try up to $m = 5$)

Generalise user-supplied $W(s)$ to $W(s, m)$

Calculate not just Q_i but Q_i^m and $\tilde{Q}_i^m$ for each slice. (Call these X_i^m and Y_i^m. Fit nicely into cartesian system)

Evaluate Δp through sum over modes. (Include r^{m-1} effects.)

– This is all done but needs tidying up.

$W(s, m)$ for simple cases found in literature.

Need to think about how to do this cleanly - also how to read optics files and decide what components are collimators and what sort. And other aspects of including different collimator shapes, e.g. scattering.

Adapting Mesh Method data

Plan as follows:

1) Run mesh code with short pulse at some radius r': look as much like a point particle as possible. This gives EM fields at any place and time.

2) Scanning values of s, calculate the longitudinal potential at some radius r and several values of ϕ.

3) Take the Fourier Transform of the ϕ dependence to get the coefficients of the $cos(m\phi)$ terms

4) This gives the $W'_m(s)$. Integrate to get the $W_m(s)$ ($\perp$ and $||$ components are related by Panofsky-Wenzel for each mode) This gives our $W(s, m)$ functions in tabulated form.

5) Try with different r and r' - should get the same answer.

What about real collimators?

All this assumes axial symmetry.

Fine for cavities and pipes. Collimators are rectangular. A slit is a special case of a rectangle.

The literature is largely silent apart from dark hints('becomes complicated' - Chao)

Can we use / adapt our approach for the general case?

What's left without axial symmetry?

Panofsky-Wenzel theorem still holds

$\frac{\partial W_\perp}{\partial z} = \nabla_\perp W_{||}$

(Proofs given often assume axial symmetry for simplicity, but it does hold generally. See Z & K p. 89)

Wake potential $\vec{W}$ still given by ∇V with some V

V is still a solution of the 2-D Laplace equation $\nabla^2_\perp V = 0$

We can solve for r and ϕ by separation of variables and again get

$V = \sum_m r^m (cos(m\phi)G(s,m) + sin(m\phi)\tilde{G}(s.m))$

For two particles, $G(s,m) = G(s,m,r',\phi')$

And we're there

According to Weiland (NIM **216** 31, (1983)) it is still true that

$$\vec{W}_\perp(r,r',\phi,\phi',s) = -e\sum m r^{m-1} r'^m W_m(s)$$
$$[\hat{r} cos m(\phi-\phi') + \hat{\theta} sin m(\phi'-\phi)]$$

Need to check and verify this. If it holds up, means that everything can be put in one framework. The only complication is that the way that the effect depends only on $\phi - \phi' \equiv \theta$ is lost - but we dropped that anyway.

Next steps

Check Weiland formula validity

Will see how examples of $W_\perp(s)$ for simple square collimators found in the literature fit into this function.

Will extend library of routines.

Will expand and clean up our Merlin code and release it to the public

Next few weeks....