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What we need for the LHC

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Terascale Physics Statistics School

DESY, March 25th 2010

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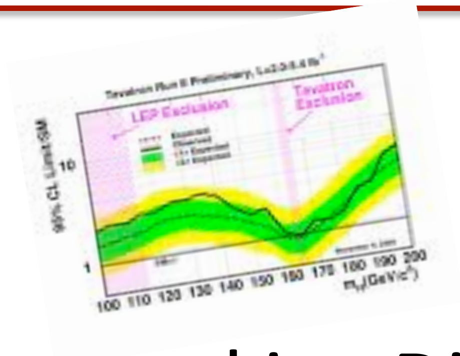
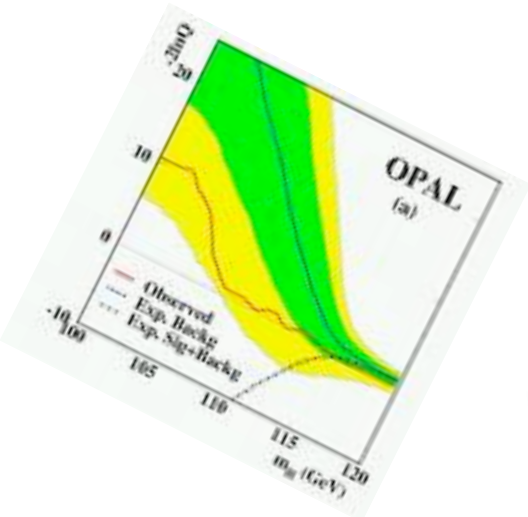
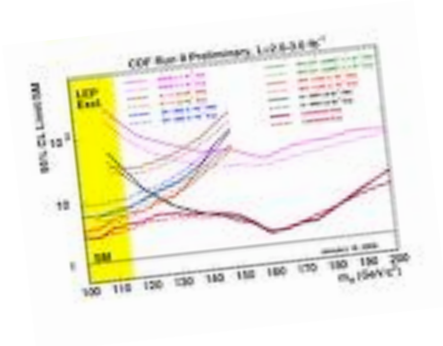
Statistics for the LHC

Not just setting limits –

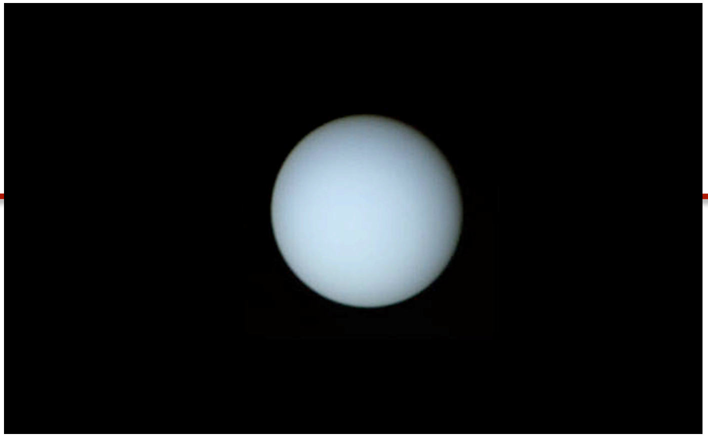






but making Discoveries

Astronomical Discovery #1



Uranus

Discovered by Herschel in 1781

An amateur astronomer – but high technology equipment

Found in a thorough survey of the sky searching for comets.

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Statistical Discovery

Significance	pValue
1σ	31.7%
2σ	4.55%
3σ	$2.70\ 10^{-3}$
4σ	$6.33\ 10^{-5}$
5σ	$5.73\ 10^{-7}$
6σ	$1.97\ 10^{-9}$

You find something weird:

- Single weird event
- Several weirdish events
- Bump in mass
- Unexpected distribution

Very unlikely that SM processes would look like this. You report *p*-value, (say 0.0027), the probability that the SM could produce an effect as weird as this – or equivalently as (in this case) a 3-sigma-effect.

Press will say "Probability that the SM could be true is only 0.27%" (or whatever)

“Probability that the Standard Model is true”

$$P(Theory \mid Data) = \frac{P(Data \mid Theory)}{P(Data \mid Theory)P(Theory) + P(Data \mid notTheory)P(notTheory)} P(Theory)$$

BAYES at work

$$P(SM \mid Data) = \frac{P(Data \mid SM)}{P(Data \mid SM)P(SM) + P(Data \mid X)P(X)} P(SM)$$

$P(SM)$ – probability that the SM is effectively true for this energy/environment

X = your favourite BSM theory. $P(Data|SM) \sim pValue$.

Presumably $P(SM) \approx 1$, $P(Data|X) \sim 1$

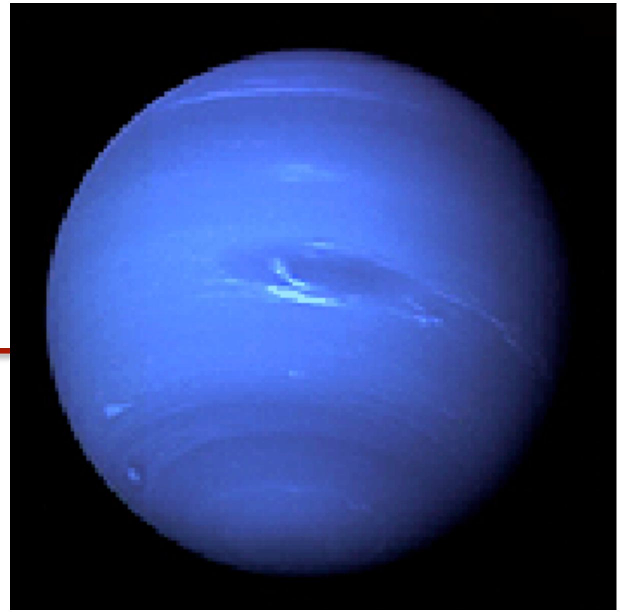
$$P(SM \mid Data) \approx \frac{pValue}{pValue + P(X)}$$

$P(X)$ is limited by $1\text{-}P(SM)$ and there are many other BSM theories.

If $P(SM)=99.9\%$ then maybe $P(X)=10^{-4}$ and $P(SM|Data)=27/28=96\%$

To knock a hole in the Standard Model, need REALLY small *p*-value

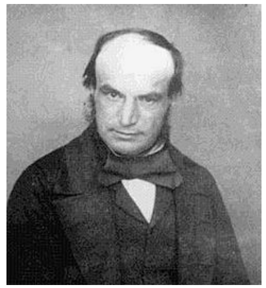
Astronomical Discovery #2



Neptune

Discovered at Berlin/Cambridge in 1846 from predictions by LeVerrier/Adams

Prediction based on discrepancies in Uranus' orbit



Observed by Galileo (and others) but not recognised for what it was

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Evaluating the p Value

Option 1:

Simulate the SM processes using Monte Carlo and count how many times this measure-of-weirdness is exceeded.

This is correct by construction (if you trust your MC). Not good for probing low-probability tails, unless you do something clever weighting events

Option 2:

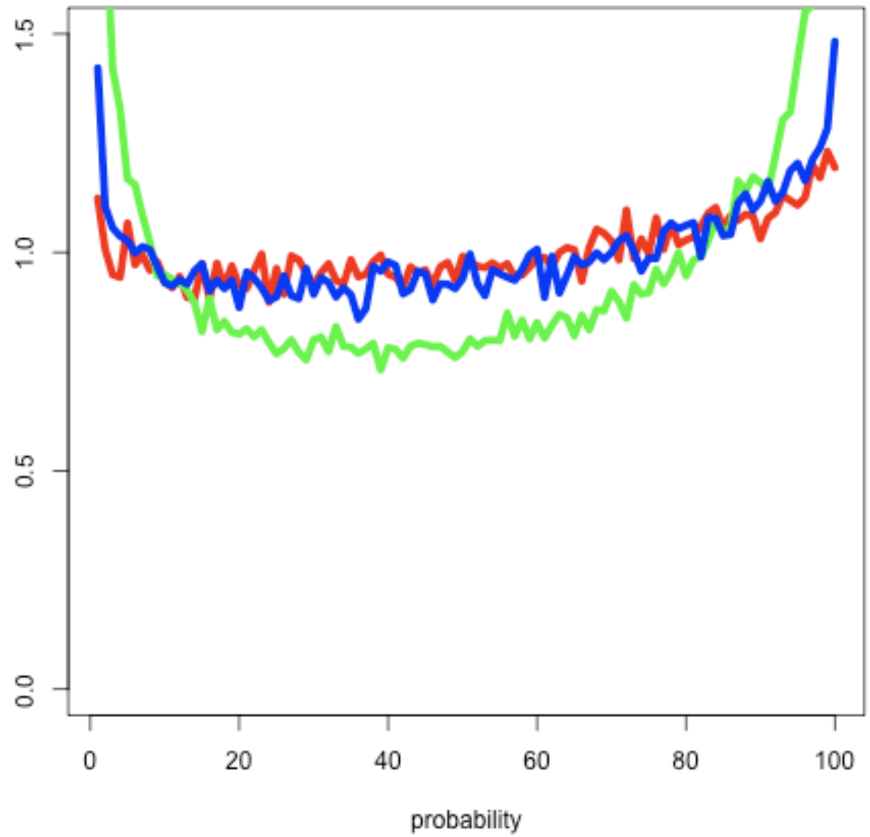
For measure-of-weirdness use a statistic with well-established mathematical properties, e.g. χ^2 distribution

Traps with χ^2

X^2 assumes Gaussian errors:

Not true for histograms, if bin contents are small

Figure shows results of toy MC simulating pValue distribution from χ^2 of histogram with ~40, 20, and 4 events/bin



χ^2 and fitting

N data points, M fitted parameters, gives χ^2 with distribution N-M 'Degrees of freedom'

Strictly speaking – only true if fitting is linear, and errors do not depend on fitted parameters. Care!

Difference of two χ^2 distributions is χ^2

If you add parameters the improvement in χ^2 tells you whether they are giving a significantly better fit (through its pValue)

But ...

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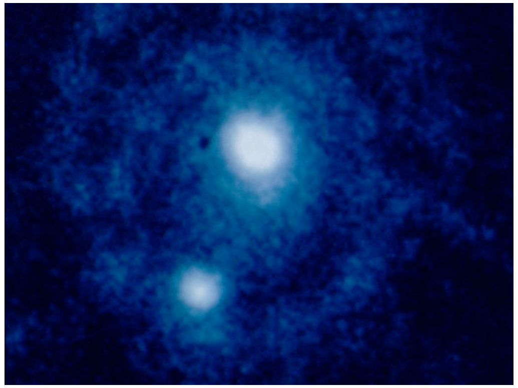
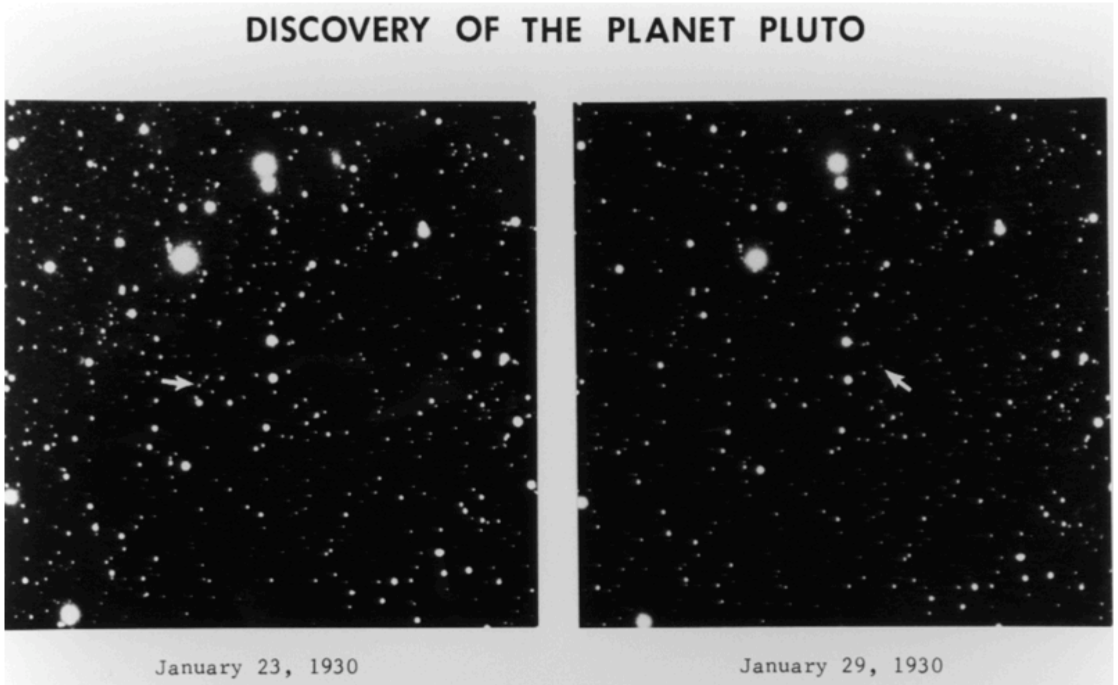
Percival Lowell

Astronomical Discovery #3

- Pluto





Discovered in 1930 by Tombaugh following predictions by Lowell based on remaining Uranus deviations

Discrepancies in Uranus' orbit now removed since better measurement of Neptune's mass

Since clear that Pluto not massive enough to be a 'planet': the Kuiper belt contains many such 'dwarf planets'

Statistics tools: another use for Maximum Likelihood

Used for parameter estimation & errors. Not for goodness-of-fit

Can be used for model comparison

For two nested models $P_0(x;a_1,a_2...a_n)$ and $P_1(x;\, a_1,a_2...a_{n+m})$, twice the improvement in Ln L is given by a χ^2 distribution with m-n degrees of freedom.

- Hard to show, but reverse obvious as Prob α $exp(-\chi^2/2)$
- Sometimes called Wilks' Theorem
- Sometime called Likelihood Ratio Test
- Subject to legal small print, e.g. samples must be large…

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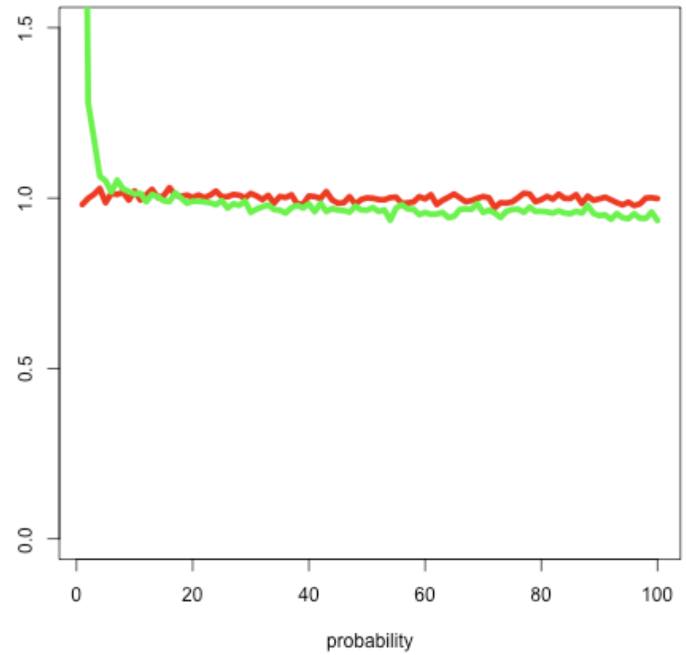
Example

Generate x in $[-0.5, 0.5]$ according to uniform distribution. $P(x)=1$

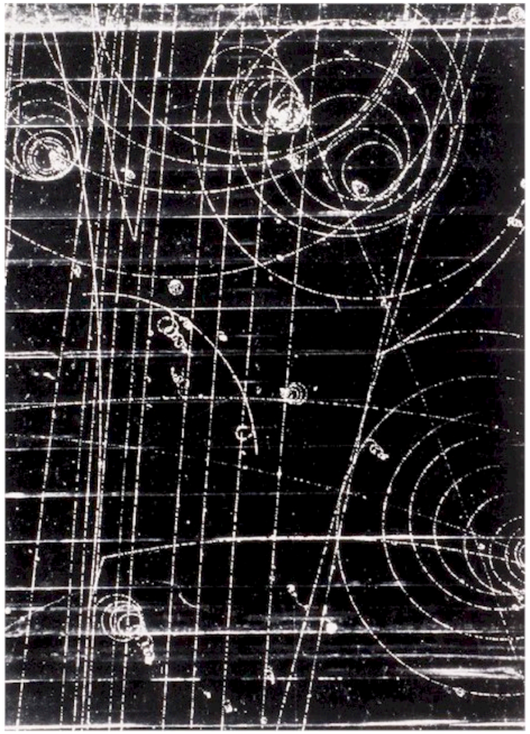
Try $P(x;a)=1+ax$

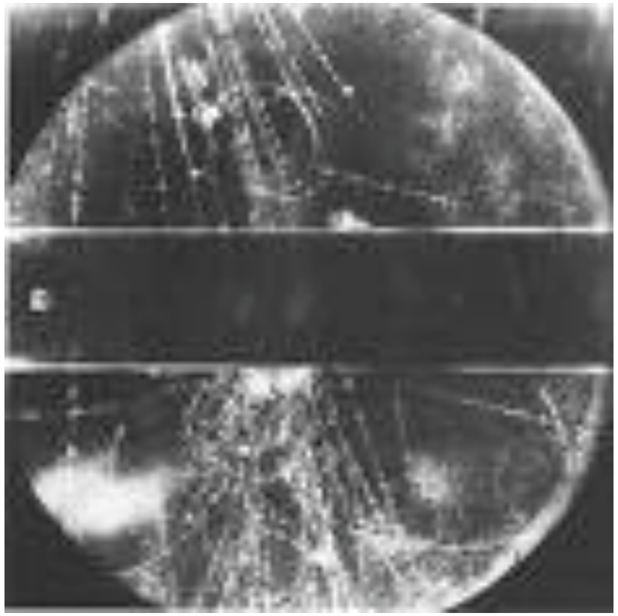
Find $\hat{a}$ using Max Likelihood and improvement in Likelihood and p-value from $Prob(2\, \Delta \ln L;1)$

Plot shows p-value distribution for 100 and for 10 x values



Two Discoveries in Particle Physics





U-type
Strange
Particles

N-type
The Ω^-

Pitfalls with $\Delta\chi^2$ and $\Delta \ln L$

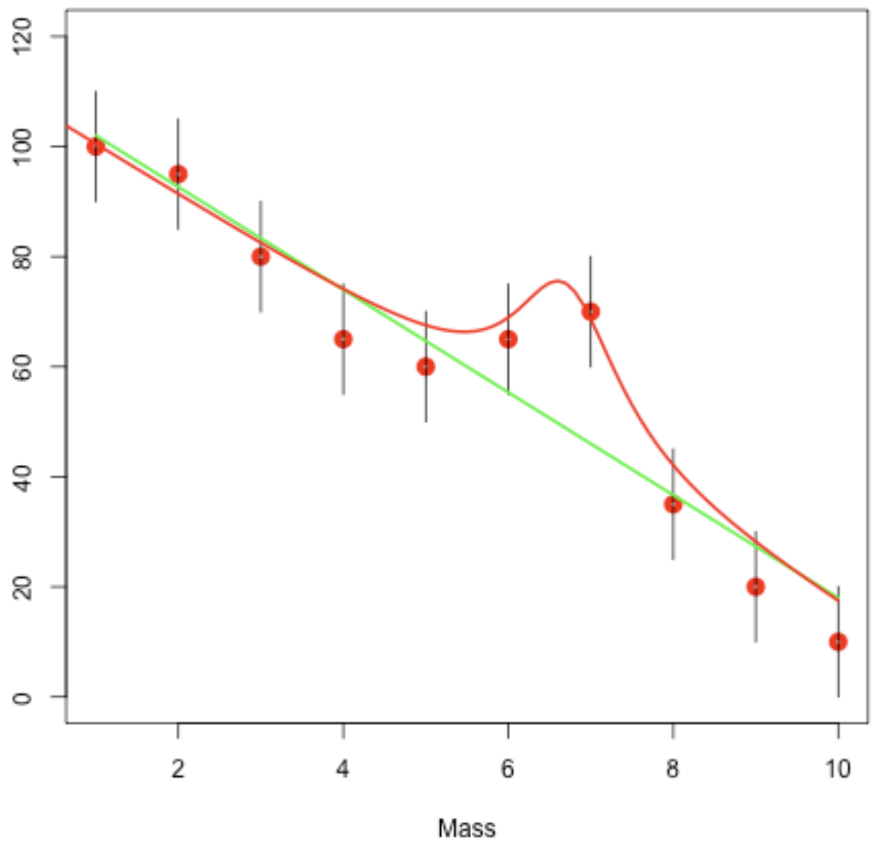
Data fitted by Background (green) or Background+signal (Red)

$Signal = N\, BW(M, M_0, \Gamma)$

Adding Signal improves χ^2 . Difference between two χ^2 values has a χ^2 distribution.

Can say - M_0, Γ fixed: Null hypothesis says improvement is χ^2 for 1 D.O.F. Prob($\Delta\chi^2$;1) gives pValue

Can't say-M_0, Γ free: Null hypothesis says improvement is χ^2 for 3 D.O.F. Prob($\Delta\chi^2$;3) gives pValue

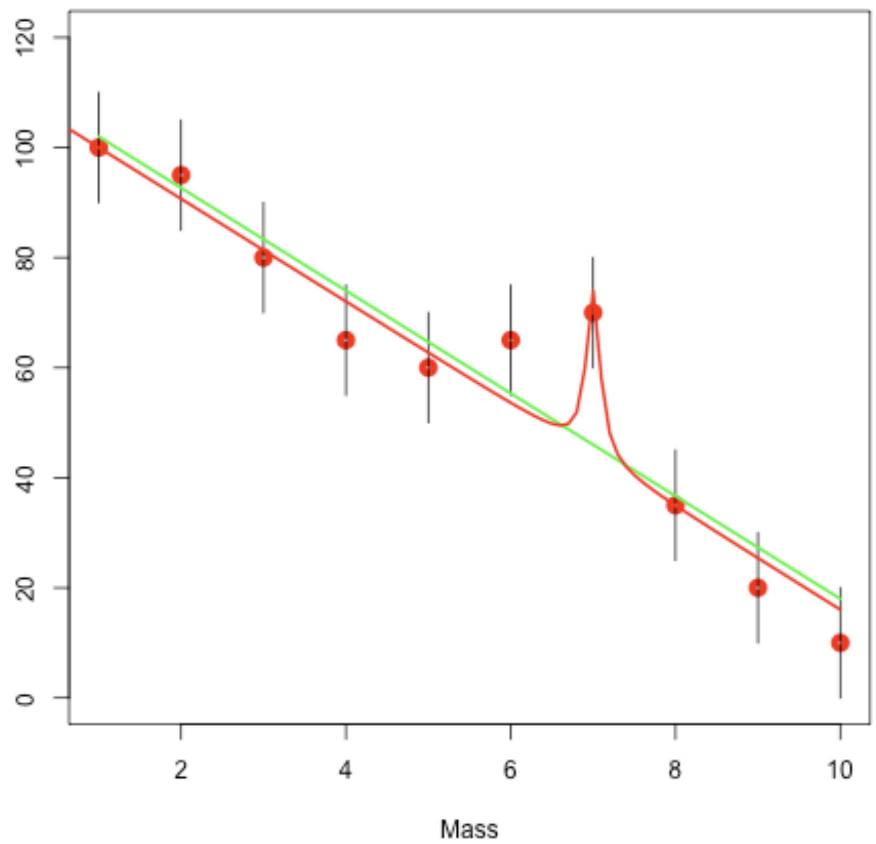


Making it obvious

For illustration, suppose Γ is fixed and small. Resonance just affects 1 bin.

If M_0 fixed then adjusting N lets you fix the value in that bin. Its contribution to χ^2 is washed out. Expected improvement 1.

If M_0 free then adjusting N lets you fix the worst bin in the plot. Expected improvement large and hard to calculate – depends on number of bins



Put like this it's obvious. Yet it goes on. Be prepared to fight your colleagues.

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Patterns of Particle Discovery

U type

Electrons

Protons

Muons

Strangeness

Ψ

D_{SJ}

N type

Positron

Gluon

W, Z

Top quark

P violation

N' type

Bottom quark

N'' type

Tau

Neutral Currents

CP violation

Dangerous Dummy Parameters

"Hypothesis testing when a nuisance parameter is present only under the alternative" - R.B. Davies Biometrika 64 p247 (1977) and 74 p33 (1987)

If the alternative 'improved' model contains parameters which are meaningless under the background-only null hypothesis then the $\Delta\chi^2$ test (etc) does not work.

Model *Background(x,a)* and *Background(x,a)+N Signal(x,a)*

Does *a* contains parameters which do not affect *Background ?*

Conclusions

- You will not find something unless you look
- What you find may not be what you're looking for.
- You need either a new technology or a prediction. Or both.
- Discovery will need hard work and perseverance
- Statistical tools will be essential, and they can be tricky

Good luck!