Beam Dynamics with MAD - Part 2

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MSc lectures

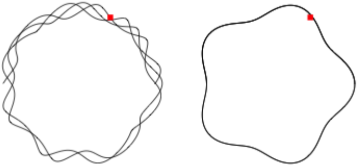
Summary

1. Resonances
2. Transverse coupling
3. Longitudinal motion
4. Dispersion
5. Chromaticity
6. Multipoles
7. Sextupoles
8. Dynamic aperture

Resonances

Sometimes called "Optical Resonances"

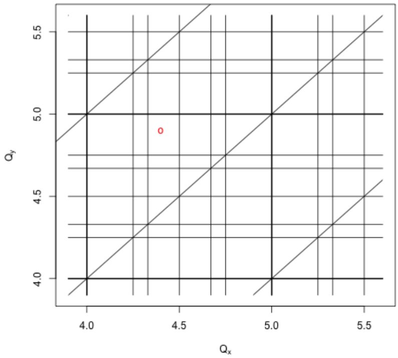
Magnets are not perfect (field strength, uniformity, position...). Consider small perturbation.



If tune Q is integer, effect acts at same point on betatron cycle. Over many turns builds up and beam lost.
Quadrupole alignment errors give resonant effect for half-integer tune.
Likewise sextupoles, octupoles, etc for $Q = N/3, N/4...$ Fraction resonances

Operating point for a ring

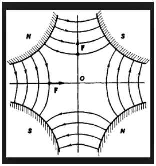
A point in Q_x, Q_y space which is safely away from resonances

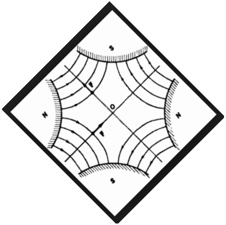


The sloping lines are caused by xy coupling. $nQ_x + mQ_y = p$ gives a resonance (n, m, p integer)

XY coupling

So far: horizontal and vertical treated as independent. $x'' = -k_x(s)x$ and $y'' = -k_y(s)y$ and 2×2 R matrices.
But actually there is coupling between horizontal and vertical motion.
Sometimes small and accidental, sometimes large and deliberate.
Solenoid. Field along z. Nonzero x' gives kick in y', and vice versa.





Skew quadrupole. Nonzero x gives kick in y', nonzero y gives kick in x'

From 2x2 to 4x4

```
S1:  SOLENOID,LENGTH=2.3,KS=0.001;
SQ1:  QUADRUPOLE, LENGTH=1.2,K1S=0.1;
SQ2:  QUADRUPOLE,LENGTH=1.2,KS=0.1,TILT=-PI/4; // SAME
```

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & K \\ 0 & 0 & 1 & 0 \\ 0 & -K & 0 & 1 \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & K & 0 \\ 0 & 0 & 1 & 0 \\ K & 0 & 0 & 1 \end{pmatrix}$$

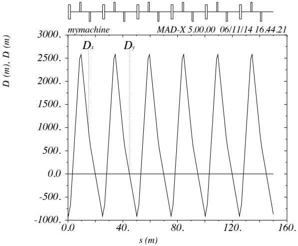
3D coupling

2+2+2=6

The 3rd direction z. MAD uses T
$T \equiv -ct$ is time wrt time of bunch at a particular s. So positive T particles are in front of the bunch.
Conjugate variable usually taken as $\delta = \Delta P/P$, i.e. fractional difference of particle momentum from nominal value. Typically $10^{-3} - 10^{-4}$.
MAD uses PT: $\Delta E/P$.
z and δ, T and PT, effect each other:
$\delta \to z$ as higher energy means higher velocity so early. Also means higher momentum and longer path so late. Competing effects: first wins at low energies, second (above γ_T) at high energies. $\mathbf{R}_{56}$
$z \to \delta$ as +ve/-ve z means particle arrives early/late at for accelerating Electric field (more next lecture). $\mathbf{R}_{65}$

Dispersion

If particle has $\Delta P > 0$, less bend from dipole magnets.
Closed orbit larger at higher energy.
Difference $\Delta x = D(s)\frac{\Delta P}{P}$
$D(s)$ the dispersion. Function of s - like β. Calculated by TWISS and can be plotted.



Matching - dispersion-free regions

Dispersion inevitable in arcs.
In straight sections (‘insertions’) may be problematic, if you need a small beam spot to make collisions. And for RF.
Design of dispersion free ('achromatic') transfer lines etc.

Chromaticity

An effect of quadrupoles

A high momentum particle ‘sees’ quadrupoles that focus more weakly → lower tune.

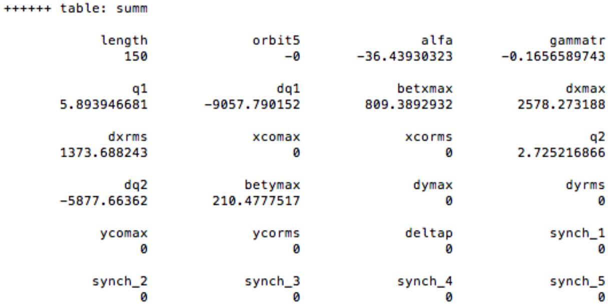
A low momentum particle ‘sees’ quadrupoles that focus more strongly → higher tune..

Spread in momentum (inevitable) means spread in tune.

Which can take you into a resonance. Bad news.

Chromaticity: $Q' = \frac{\partial Q}{\partial(\Delta p/p)} \qquad \xi = \frac{Q'}{Q}$

Look at the TWISS output



```
++++++ table: summ

        length              orbit5                alfa             gammatr
           150                  -0        -36.43930323       -0.1656589743

            q1                 dq1             betxmax               dxmax
   5.893946681        -9057.790152         809.3892932         2578.273188

         dxrms              xcomax              xcorms                  q2
   1373.688243                   0                   0         2.725216866

           dq2             betymax               dymax               dyrms
   -5877.66362         210.4777517                   0                   0

        ycomax              ycorms              deltap             synch_1
             0                   0                   0                   0

       synch_2             synch_3             synch_4             synch_5
             0                   0                   0                   0
```

length of orbit
cT of orbit
momentum compaction
transition energy
horizontal tune
chromaticity
max β
max dispersion
rms dispersion
closed/ref orbit differenc
etc

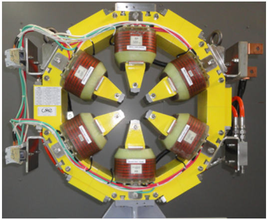
Multipole magnets

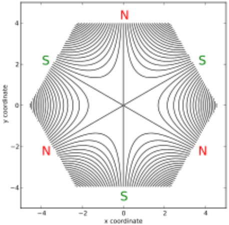
A brief diversion....

Question: What is $\vec{B}(\vec{r})$?
Write $\vec{B} = -\nabla\phi$ as this is magnetostatics
Inside magnet, z irrelevant and $B_z = 0$ so problem is 2D
Question becomes: What is $\phi(\vec{r}) = \phi(r, \theta)$?
Fourier expansion $\phi = \sum_1^\infty A_n(r)cos(n\theta) + B_n(r)sin(n\theta)$
Maxwell: $Div\vec{B} = 0$ so $\nabla^2\phi = 0$
$(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2})\phi = 0$ applies term-by-term
Solution $A_n(r) = a_n r^n, B_n(r) = b_n r^n$ (discard r^{-n})
So $\vec{B}(\vec{r})$ specified by $a_1, b_1, a_2, b_2, a_3, b_3...$
Normal (a) and Skew (b) Multipoles
To get from number to name, multiply by 2 and translate into Latin
A quadrupole magnet (i.e. 4 poles) given a quadrupole field - plus, maybe, higher terms.

Fixing the chromaticity

Sextupole Magnets





$B_y = \frac{1}{2} B_2 (x^2 - y^2) \qquad B_x = B_2 xy$

Acts like a quadrupole with strength proportional to distance

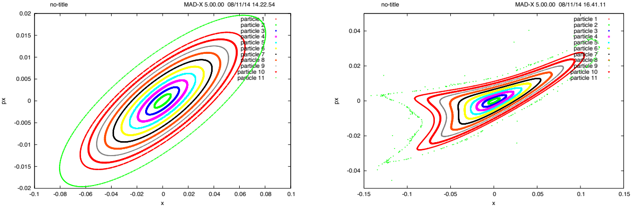
Insert at point with large dispersion, so strength $\propto$ distance $\propto \delta$

Use to cancel quadrupole chromaticity

Dynamic aperture

Aperture: particles with large displacement hit the beam pipe and are lost.

```
TRACK,file=track,dump;
n=1;
while(n<12) {START,PX=0,X=N*4.E-3; n=n+1;}
RUN,TURNS=1000;
ENDTRACK;
```



Phase space for particles in a lattice (a) normal (b) with a sextupole
Dynamic aperture: particles with large displacement develop chaotic motion due to nonlinear sextuples.

Assessment

Using the same ring as specified in the last assignment, after the matching.
Plot the dispersion functions around the ring.
Add a sextuple to the ring. Use MATCH to adjust its field strength to set the horizontal chromaticity to 0. What is that field strength (in real units)?
Use TRACK and PLOT to investigate the dynamic aperture.
Remove the sextuple and add a new sextuple in each sector (10 altogether). Again, use MATCH to set the horizontal chromaticity to zero. What is the field strength?
Use TRACK and PLOT to investigate the dynamic aperture.
Your answer should be submitted on UniLearn as a single document including your MAD commands, selected output, the requested plots, and the necessary words of explanation. I will expect to be able to run your MAD commands.