Beam Dynamics with MAD - Part 5

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Insertions

Accelerating particles is not an end in itself.

- Injection
- Extraction
- Interaction Regions
- Synchrotron Radiation

and to do all this we need **bumps**

Extraction

Note: Extraction and Injection similar - in complex systems extraction from one ring may be injection into another.

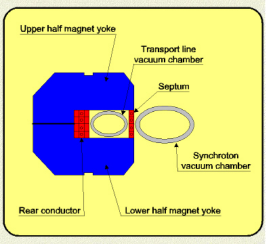
Particles in stable orbit on turn N, deflected into some other path for turn $N+1$.

Not easy - thanks to Liouville's theorem and Maxwell's equations.

Usually Fast Extraction: whole bunch is affected. There is also slow, multi-turn, extraction which removes a few particles at a time..

Technologies: kicker and septum magnets

Kicker magnet: dipole that can be switched on in time (typically ns) between bunches. Pulse itself also short ($ns \rightarrow \mu s$). Low impedance (no iron - maybe ferrite), massive current (kA), fast switch (thyratron)



Septum magnet: Thin sheet of conductor, bearing current, so magnetic field different on both sides. (There are also electrostatic septa)

Extraction: Kicker followed by septum

Injection Regions

Same as extraction but backwards - up to a point.

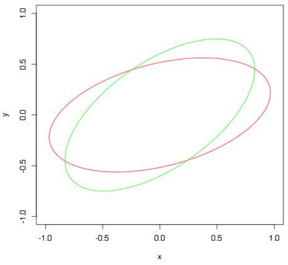
Septum so that injected and 1st turn bunches have close paths

Kicker to put injection bunch onto 1st turn orbit, then vanish before next orbiting bunch.

Useful trick: charge exchange. Accelerate H^- ions, pass through foil which strips to H^+

N.b. can only be played once.

Filamentation



Get things wrong on injection and the emittance increases (despite Liouville)

Remember: Twiss parameters of the machine define an elliptical shape. Area (=emittance) is arbitrary.

Turn by turn, particles move round their ellipse, so shape of bunch is the same.

Injected bunch also has elliptical shape defined by α, β, γ. If this is the same ('matches') accelerator, all well.

If not then particles move round on their new ellipses - 'filaments'

Liouvillle conserved as area of twisty shape = area of original ellipse - but after several turns new shape is effectively larger ellipse.

Interaction Regions

Colliding particles in contra-rotating beams.

May use 2 rings (LHC, BaBar, HERA...) or 1 ring with particles and antiparticles (LEP, Tevatron...)
Collision rate at interaction point (IP) is $\sigma\mathcal{L}$, where $\mathcal{L}$ is the Luminosity

$$\mathcal{L} = \frac{N_b f\, N_1 N_2}{4\pi\sigma_x^*\sigma_y^*}$$

for N_b bunches at orbit frequency f, with N_1 and N_2 particles per bunch, on average, and rms spreads σ_x^*, σ_y^*. (* denotes quantities at the IP.) Units $cm^{-2}\,s^{-1}$. Typical values 10^{30+}

From Twiss properties, $\sigma = \sqrt{\beta\epsilon}$. So want ϵ and β to be as small as possible ('squeeze the beams')

Near the IP, $\beta(s) = \beta^* + \frac{s^2}{\beta^*}$. The nearest quad may be far away

Synchrotron Radiation

1. Accelerated charges emit EM radiation
2. Acceleration refers to velocity, not speed.
3. Changing the direction of a particle is a change in its velocity
4. Hence particles in magnetic fields emit EM radiation

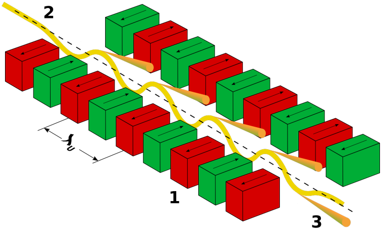
Process a problem for particle physics but useful for X ray (and other) diffraction.
Rate of emission $P = \frac{e^2 c}{6\pi\epsilon_0} \frac{1}{(m_0 c^2)^4} \frac{E^4}{R^2}$ so most important for electrons.
Quantum process. Often best to think of it in terms of photons.

4 Generations of Synchrotron light sources

1. As a by-product of bending magnets in other accelerators
2. From bending magnets in dedicated accelerators (SRS at Daresbury)
3. Wigglers and Undulators in straight sections (Diamond)
4. Free electron lasers (LCLS, Tesla,...)

Wigglers and undulators

Succession of opposing dipole fields in a straight section



Wigglers: EM radiation from each bend separately
Undulators: EM radiation from each bend coherently. Constructive interference at right match of angle, wavelength, and magnet spacing. Give narrower spectrum and angular spread than wigglers.
Free Electron Laser (FEL): constructive interference between radiation from individual electrons. Needs low ϵ and high β

Vital tool for insertions: local orbit bumps

Aim: to shift the position (or direction) of the beam without disturbing the rest of the ring. Usually in the horizontal direction, but can be vertical

Examples

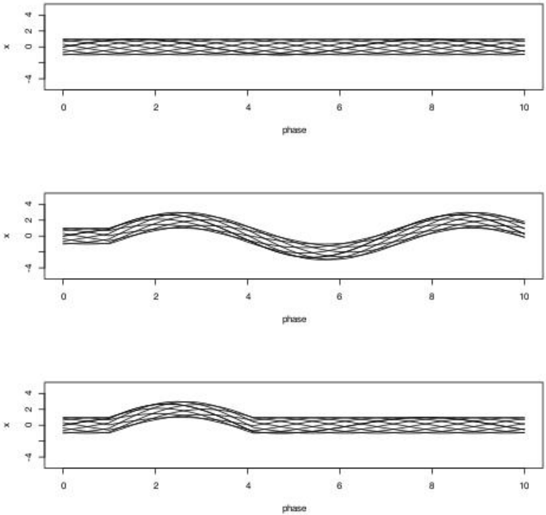
- Painting on injection: move phase space of beam around.
- Align particles for wigglers/undulators
- Move intersecting beams from just-miss to full collision

Use: corrector magnets (a.k.a. kickers, but these are slow) spread out among all the others.

One magnet is not enough. It just excites betatron oscillations.

Two magnets is possible. Positions must differ π in betatron phase and strengths in ratio $\frac{K_1}{K_2} = \sqrt{\frac{\beta_2}{\beta_1}}$. Called the 180° bump and not very common, but varying K_1 means you can vary the mean displacement in x by any desired amount x Three of four magnet bumps are more flexible and more usual.

The 180° bump



Plot shows position in scaled space - to get real displacement multiply by factor $\sqrt{\epsilon\beta(s)}$,

To get real distance s (locally) multiply by $\beta(s)$

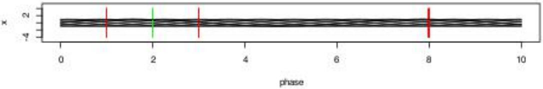
Top plot: no kick, just envelope

Middle plot: kick applied

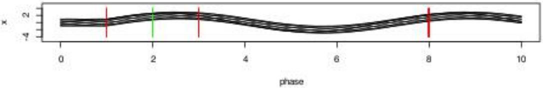
Bottom plot: second equal kick applied π later

The three-magnet bump

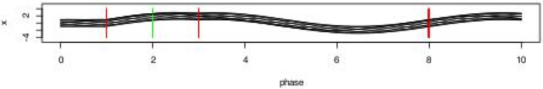
See Klaus Wille, *The Physics of Particle Accelerators*, section 3.18



Three magnets at arbitrary s (i.e. phase)



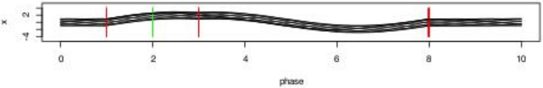
First magnet gives kick K for desired displacement



Second magnet kick

$$-\sqrt{\frac{\beta_1}{\beta_2}}\frac{\sin(\Psi_3-\Psi_1)}{\sin(\Psi_3-\Psi_2)}K$$

puts beam on axis at 3rd



Third magnet kick

$$-\sqrt{\frac{\beta_1}{\beta_3}}\left(\frac{\sin(\Psi_3-\Psi_1)}{\tan(\Psi_3-\Psi_2)}-\cos(\Psi_3-\Psi_1)\right)K$$

Where did all that come from?

2 stages

The transfer matrices

Suppose **A** transports $\begin{pmatrix} x \\ x' \end{pmatrix}$ from magnet $1 \to 3$ and **B** from $2 \to 3$.

The system is linear, so the effect at magnet 3 of the kicks K_1 and K_2 is

$$\mathbf{A}\begin{pmatrix} 0 \\ K_1 \end{pmatrix} + \mathbf{B}\begin{pmatrix} 0 \\ K_2 \end{pmatrix} = \begin{pmatrix} A_{12}K_1 + B_{12}K_2 \\ A_{22}K_1 + B_{22}K_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -K_3 \end{pmatrix}$$

which you solve for K_2 and K_3.

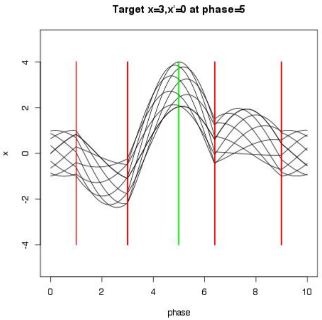
The link from matrices to phase

Matrix can be written in terms of Twiss parameters and phase difference Ψ

$$\mathbf{M_{1\to 2}} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\Psi + \alpha_1 \sin\Psi) & \sqrt{\beta_1\beta_2}\sin\Psi \\ \frac{1}{\sqrt{\beta_1\beta_2}}\left((\alpha_1 - \alpha_2)\cos\Psi - (1+\alpha_1\alpha_2)\sin\Psi\right) & \sqrt{\frac{\beta_1}{\beta_2}}(\cos\Psi - \alpha_1\sin\Psi) \end{pmatrix}$$

In this scaled space, β is constant and α vanishes and this is just a rotation. In real world full detail is included.

Four-magnet bumps



Kicks of magnets 1 and 2 adjusted to give desired displacement X and direction X' at specified point P:

$$\mathbf{M_{1\to P}} \begin{pmatrix} 0 \\ K_1 \end{pmatrix} + \mathbf{M_{2\to P}} \begin{pmatrix} 0 \\ K_2 \end{pmatrix} = \begin{pmatrix} X \\ X' \end{pmatrix}$$

Kicks of magnets 3 and 4 found using reflection of $\mathbf{M_{p\to 3}}$ and $\mathbf{M_{p\to 4}}$

Formulæ in Wille 3.16

Problem

Set up a transfer line of length to be assigned, consisting of 4 F0D0 pairs. Plot the beta functions.
Introduce a 3-magnet bump that will move the horizontal co-ordinate at the point halfway along the line by 1 cm horizontally from the centre, without disturbing the trajectory outside the 3 magnets. Use MATCH to select the kicks required.
What is the transverse momentum you obtain at this halfway point?
You will find Werner Herr's *MAD-X primer* helpful for this problem.