Introduction to Systematic Errors

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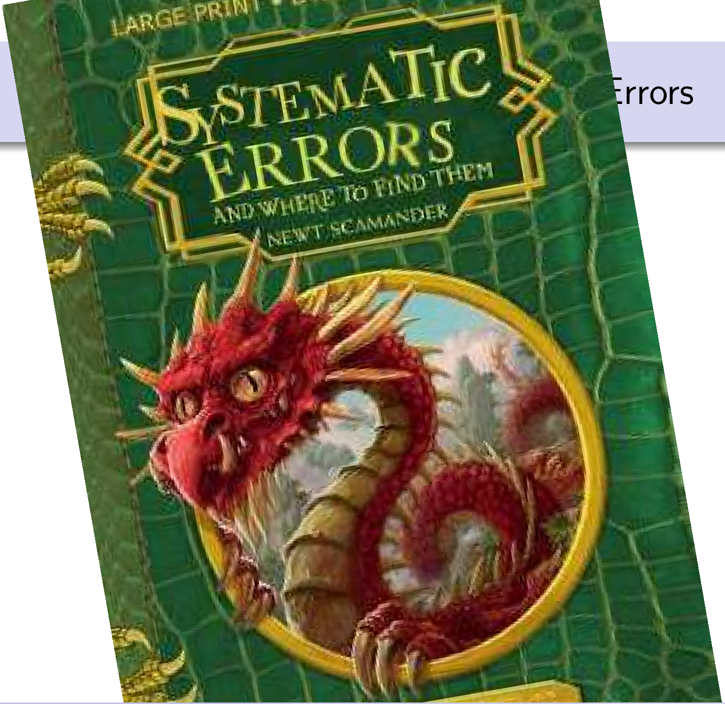
Errors


LARGE PRINT
SYSTEMATIC ERRORS
AND WHERE TO FIND THEM
NEWT SCAMANDER

Systematic Errors

I've given this talk (in various versions) many times.

Originally it was controversial and radical.

Today its views have been widely accepted - but they still need to be said.

There is still a lot of bad practice out there. Muddled thinking and following traditional procedures without understanding them.

When statistical errors dominated, this didn't matter much. In the days of particle factories and big data samples, it does.

People are scared of systematic errors because they are ignorant - ignorance leads to fear... They follow familiar rituals they hope will keep them safe.

The talk covers...

- What is a Systematic Error?
- How to deal with them
- How to evaluate them
- Checking your analysis
- Conclusions and recommendations

What is a Systematic Error?

Systematic error: reproducible inaccuracy introduced by faulty equipment, calibration, or technique.

Bevington: Data Reduction and Error Analysis

Systematic effects is a general category which includes effects such as background, scanning efficiency, energy resolution, variation of counter efficiency with beam position and energy, dead time, etc. The uncertainty in the estimation of such a systematic effect is called a systematic error.

Orear: . Notes on Statistics for Physicists

These are contradictory

Orear is RIGHT

Bevington is WRONG

So are a lot of other books and websites

An error is not a mistake

We teach undergraduates the difference between *measurement errors*, which are part of doing science, and *mistakes*.

If you measure a potential of 12.3 V as 12.4 V, with a voltmeter accurate to 0.1V, that is fine. Even if you measure 12.5 V

If you measure it as 124 V, that is a mistake.

Bevington is describing *systematic mistakes*

Orear is describing *systematic uncertainties* - which are 'errors' in the way we use the term.

Avoid using 'systematic error' and always use 'uncertainty' or 'mistake'? Probably impossible. But should always know which you mean

Examples and two key facts

Track momenta from $p_i = 0.3B\rho_i$ have statistical errors from ρ and systematic errors from B

Calorimeter energies from $E_i = \alpha D_i + \beta$ have statistical errors from light signal D_i and systematic errors from calibration α, β

Branching ratios from $Br = \frac{N_D - B}{\eta N_T}$ have statistical error from N_D and systematic errors from efficiency η, background B, total N_T

Nuisance Parameters

is a useful way to think about systematic uncertainties. Parameters which are unknown and hence uncertain, but that you're not interested in.

More Data

Taking more measurements and averaging does not reduce the error.

Goodness of fit

There is no friendly χ^2 test to warn you if your error estimation is wrong

Bayesian or Frequentist?

Can be either

Frequentist: Errors determined by an *ancillary experiment* (real or simulated)

E.g. magnetic field measurements, calorimeter calibration in a testbeam, efficiency from Monte Carlo simulation

Sometimes the ancillary experiment is also the main experiment - e.g. background from sidebands.

Bayesian: theorist thinks the calculation is good to 5% (or whatever). Experimentalist affirms calibration will not have shifted during the run by more than 2% (or whatever)

Some analysis techniques use hybrid of frequentist and Bayesian.

How to handle them: correlation

Systematic uncertainties obey the same rules as statistical uncertainties

We write $x = 12.2 \pm 0.3 \pm 0.4$ "where the first is the statistical and the second is the systematic error", but we could write $x = 12.2 \pm 0.5$.
For single measurement extra information is small.

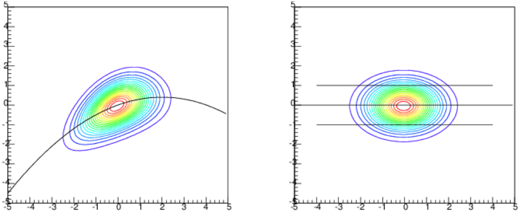
For multiple measurements e.g. $x_a = 12.2 \pm 0.3, x_b = 17.1 \pm 0.4, all \pm 0.5$ extra information important, as results correlated.
Examples: cross sections with common luminosity error, branching ratios with common efficiency ...

Use standard combination of errors formula, including the correlation term

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + 2\rho \left(\frac{\partial f}{\partial x}\right) \left(\frac{\partial f}{\partial y}\right) \sigma_x \sigma_y$$

Non-Gaussian errors 1

Profile Likelihood - motivation (not very rigorous)



You have a 2D likelihood plot with axes a_1 and a_2. You are interested in a_1 but not in a_2 ('Nuisance parameter')

Different values of a_2 give different results (central and errors) for a_1

Suppose it is possible to transform to $a_2'(a_1, a_2)$ so L factorises, like the one on the right. $L(a_1, a_2') = L_1(a_1)L_2(a_2')$

Whatever the value of a_2', get same result for a_1

So can present this result for a_1, independent of anything about a_2'.

Path of central a_2' value as fn of a_1, is peak - path is same in both plots

So no need to factorise explicitly: plot $L(a_1, \hat{\hat{a}}_2)$ as fn of a_1 and read off 1D values.

$\hat{\hat{a}}_2(a_1)$ is the value of a_2 which maximises $\ln L$ for this a_1

Non-Gaussian errors 2

Marginalised likelihoods

Instead of profiling, just integrate over a_2.

Can be very helpful alternative, specially with many nuisance parameters

But be aware - this is strictly Bayesian

Frequentists are not allowed to integrate likelihoods wrt the parameter

$\int P(x; a)\, dx$ is fine, but $\int P(x; a)\, da$ is off limits

Reparametrising a_2 (or choosing a different prior) will give different values for a_1

Evaluating Systematic Errors in your analysis

3 types

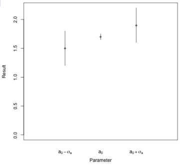
1) Uncertainty in an explicit continuous parameter:

E.g. uncertainty in efficiency, background and luminosity in branching ratio or cross section

Standard combination of errors formula and algebra, just like undergraduate labs. Have to include correlations but this is all handled by matrices.

Evaluating Systematic Errors (2)

Uncertainty in an implicit continuous parameter such as: MC tuning numbers (σ_{p_T}, polarisation......)



Not amenable to algebra

Method: vary parameter by $\pm\sigma$ and look at what happens to your analysis result (directly, or through efficiency, background etc.)

Note 1: Hopefully effect is equal but opposite - if not then can introduce asymmetric error, but avoid if you can. Rewrite $^{+0.5}_{-0.3}$ as ± 0.4

Note 2. Your analysis results will have errors due to e.g. MC statistics. Some people add these (in quadrature). This is wrong. Technically correct thing to do is subtract them in quadrature, but this is not advised.

Note 3: Or take many Gaussian samples of parameter value and look at distribution of result. Nice, if you have the computing capacity.

Evaluating Systematic Errors (3)

Discrete uncertainties, typically in model choice

Situation depends on status of model. Sometimes one preferred, sometimes all equal (more or less)

With 1 preferred model and one other, quote $R_1 \pm |R_1 - R_2|$

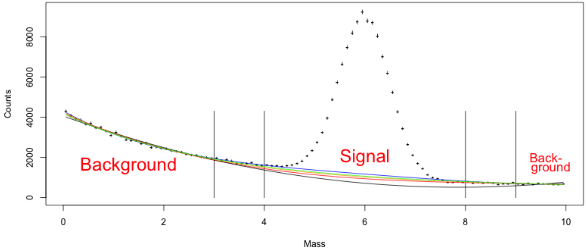
With 2 models of equal status, quote $\frac{R_1+R_2}{2} \pm |\frac{R_1-R_2}{\sqrt{2}}|$

N models: take $\overline{R} \pm \sqrt{\frac{N}{N-1}(\overline{R}^2 - \overline{R}^2)}$ or similar mean value

2 extreme models: take $\frac{R_1+R_2}{2} \pm \frac{|R_1-R_2|}{\sqrt{12}}$

These are just ballpark estimates. Do not push them too hard. If the difference is not small, you have a problem - which can be an opportunity to study model differences.

Example: many models



Analysis

Count number of events in signal region.
Subtract background by fitting quadratic in background region (red curve)

Systematics

Try using cubic, quartic, exponential, exponential-quadratic ... functions to fit background

Points to note

- The background function has no theoretical grounding. It just fits the data.
- This is in addition to the contribution from the uncertainty in the fitted function parameters
- Sometimes signal and background regions can be in different channels/experiments, but same logic applies.
- For the sake of the example, we suppose all functions give acceptable fits to data in the background region.
- If you know the likelihood function (including the peak) you can use the profile+envelope method [1], but not if you're just counting numbers

[1] P. D. Dauncey, M. Kenzie, N. Wardle and G. J. Davies, Journal of Instrumentation 10 p04015 (2015), arXiv:1408.6865v5

Possible choices

For your central analysis value, you can stick with your favourite or you can use the average of them all, depending on what you think of their status.

For the uncertainty you can take the RMS deviation of the values about the chosen central value. Or the mean absolute deviation.

If you take it as the maximum deviation then this is the wrong choice.

Why this is wrong

- It is over-conservative and inflates your errors
- It penalises diligence: if you consider many functions you are bound to make your errors larger

Why do people do it?

From a recent LHCb PhD thesis

10.2.3 Barrier factors

The radii of the Blatt-Weisskopf barrier factors are fixed to $1.6\,\mathrm{GeV}^{-1}$ in the fit for the amplitude model. A set of fits are performed varying these values between $1.0\,\mathrm{GeV}^{-1}$ and $2.5\,\mathrm{GeV}^{-1}$ with the largest deviation taken to be the systematic uncertainty.

10.2.4 Alternative lineshapes

The lineshapes chosen to model some of the components of the fit are sources of systematic uncertainty and so alternatives are tried for some of them. In particular, new models are tried for the $\pi\pi$ S-wave, the $D\pi$ S-wave and the ρ–ω mixing component. Again the greatest deviation is taken to be the systematic uncertainty.

"It's what we always do"

That is not a valid reason for doing anything

"If the result turns out to be outside the quoted error will be bad for my/our reputation"

32 % of our results should be outside their quoted error

"It's conservative"

For errors, *conservative* is another word for *wrong*

Checking the analysis

"As we know, there are known knowns. There are things we know that we know. There are known unknowns. That is to say, there are things that we know we don't know. But there are also unknown unknowns. There are things we don't know we don't know."

Donald H Rumsfeld

Checking the analysis: Errors are not mistakes - but mistakes still happen.

Statistical tools can help find them - though not always give the solution.

- Check by repeating analysis with changes which *should* make no difference:
  - Data subsets
  - Magnet up/down
  - Different selection cuts
  - Changing histogram bin size and fit ranges
  - Changing fit technique
- Looking for impossibilities

Example: the BaBar CP violation measurement ".. consistency checks, including separation of the decay by decay mode, tagging category and B_{tag} flavour... We also fit the samples of non-CP decay modes for $\sin 2\beta$ with no statistically significant difference found."

If it passes the test

Tick the box and move on

Do not add the discrepancy to the systematic error

- It's illogical
- It penalises diligence
- Errors get inflated

The more tests the better. You cannot prove the analysis is correct. But the more tests it survives the more likely your colleagues[2] will be to believe the result.

[2] and eventually even you

If it fails the test

Worry!

- Check the test. Very often this turns out to be faulty.
- Check the analysis. Find mistake, enjoy improvement.
- Worry. Consider whether the effect might be real. (E.g. June's results are different from July's. Temperature effect? If so can (i) compensate and (ii) introduce implicit systematic uncertainty)
- Worry harder. Ask colleagues, look at other experiments

Only as a last resort, add the term to the systematic error. Remember that this could be a hint of something much bigger and nastier

Clearing up a possible confusion

What's the difference between?

Evaluating implicit systematic errors: vary lots of parameters, see what happens to the result, and include in systematic error

Checks: vary lots of parameters, see what happens to the result, and don't include in systematic error

(1) Are you expecting to see an effect? If so, it's an evaluation, if not, it's a check

(2) Do you clearly know how much to vary them by? If so, it's an evaluation. If not, it's a check.

Cover cases such as trigger energy cut where the energy calibration is uncertain - may be simpler to simulate the effect by varying the cut.

So finally:

1. Thou shalt never say ‘systematic error’ when thou meanest ‘systematic effect’ or ‘systematic mistake’.
2. Thou shalt know at all times whether what thou performest is a check for a mistake or an evaluation of an uncertainty.
3. Thou shalt not incorporate successful check results into thy total systematic error and make thereby a shield to hide thy dodgy result.
4. Thou shalt not incorporate failed check results unless thou art truly at thy wits’ end.
5. Thou shalt not add uncertainties on uncertainties in quadrature. If they are larger than chickenfeed thou shalt generate more Monte Carlo until they shrink to become so.
6. Thou shalt say what thou doest, and thou shalt be able to justify it out of thine own mouth; not the mouth of thy supervisor, nor thy colleague who did the analysis last time, nor thy local statistics guru, nor thy mate down the pub.

Do these, and thou shalt flourish, and thine analysis likewise.